

What is Motion?

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Zeno of Elea is famous for his arguments against motion and plurality. Of his many arguments, only fragments of a very few remain. This is unfortunate, for we can only speculate about those that have been lost. The fragments that have come down to us display what appear to be somewhat simpleminded arguments against space and motion, and many have applied modern scientific theories and techniques of mathematics and set theory to find solutions to his paradoxes. However, I think that these “solutions” miss the point; Zeno’s arguments do not avail themselves of mathematical solutions, and physics provides no clues as to what motion actually is. My goal, therefore, is to construe these arguments in the best possible light, and, building one upon the other, show that we are no closer to understanding motion now than were the ancient Greeks over 2,400 years ago.

The first argument that we shall consider takes place on a race course. Zeno attempts to show that it is impossible for an object to move from one place to another, because it must first reach the halfway point, and yet before that it must reach the halfway point to the halfway point, and so on *ad infinitum*. Assuming that space is infinitely divisible, any given distance, however small, must have a midpoint. Furthermore, any given distance, however small, must take some amount of time to cross. Thus there is no motion, since it is impossible for an object to traverse an infinite number of distances in a finite time. (Baird and Kaufmann, 24-5)

Now, it must be admitted that this argument is not very compelling. The obvious solution, and the one that occurred to Aristotle, is that it takes a shorter amount of time to cross a shorter

distance than it does to cross a longer one. And we can easily show, without recourse to calculus, that an infinite sum of fractions does indeed add up to a finite number:

$$\begin{array}{ll}
 S = \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots\right) & \text{definition} \\
 S = \frac{1}{2} + \left(\frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots\right) & \text{pull } \frac{1}{2} \text{ out from parentheses} \\
 2S = 1 + \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots\right) & \text{multiply both sides by two} \\
 2S = 1 + S & \text{substitution} \\
 S = 1 & \text{subtract } S \text{ from both sides}
 \end{array}$$

Repeatedly subdividing a finite interval into halves generates the infinite series S , and as has been demonstrated, this infinite series adds up to the whole of the finite interval. But what does this tell us, beyond the obvious point that the parts add up to the whole? Utilizing more sophisticated mathematical methods in calculus and set theory takes us no further than this simple fact. So, just as an infinite series of fractional distances can add up to a finite distance, so an infinite series of fractional times can add up to a finite time interval. And since it takes only a fractional time to cross a fractional distance, and the cardinality of both series is equal, it seems that we can indeed have motion. This is most obvious in modern times, as the equation for velocity in classical mechanics is $v = d/t$. If the velocity is constant, then a shorter distance to cross necessarily means that it will be crossed in a shorter time. But this solution only works if we admit that, in regard to motion, space and time are inextricably linked. Space and time are one entity: space-time.

The second argument has Achilles running along a racecourse, trying to catch up to a turtle that has a head start. Naturally, Achilles runs much faster than the turtle, and should be able to catch it very quickly. The problem is that, however fast Achilles runs, by the time he reaches the point where the turtle *was*, the turtle has moved some distance forward, and so on *ad*

infinitum. Thus, concludes Zeno, Achilles will never catch the turtle. (Baird and Kaufmann, 25-6)

Mathematically, we can represent the race course as a number line, representing Achilles by 'A', the turtle by 'B', their starting points being a and b respectively, and moving at respective velocities V_A and V_B . We can then construct the following system of equations, representing by 'c' the point at which Achilles would catch the turtle:

$$V_A = (c-a)/t$$

$$V_B = (c-b)/t$$

Where $t > 0$, $c > b > a > 0$, and $V_A > V_B$.

Assuming that Achilles and the turtle are moving at fixed velocities, we can solve this system of equations for the point c at which Achilles will catch up to the turtle. This "solution", however, misses the point; in fact, it constitutes a refusal to deal with the problem that Zeno raises. The question is not whether Achilles and the turtle can cross two distances of differing length in the same time interval, but whether or not Achilles can ever actually "catch up" to the turtle. To solve this problem, we must consider the (presumably) decreasing interval between Achilles and the turtle. So long as there remains some distance between them, Achilles can only catch up if the turtle does not move forward (otherwise there would be yet another, albeit smaller, interval between them). What there must be, therefore, is some very small discrete interval wherein we can say that Achilles moves, and the turtle does not; that would be the interval in which Achilles catches the turtle. What we must say, therefore, is that, relative to two bodies in motion, there is some smallest meaningful interval of space-time.

The third argument we shall consider is a very simple one indeed. We imagine an arrow in flight. But at any given instant during its flight, the arrow is exactly where it is, and nowhere

else. What we notice is that the arrow, considered in and of itself, is motionless. (Baird and Kaufmann, 26)

Here again mathematics fails us. In the Calculus, there is indeed a notion of “instantaneous velocity”: it is a convenient term for the slope of the tangent at some particular point along a curve, which in turn represents the velocity of some object. Since this point is an instant in time (i.e. there is no elapse in time), and there is no change in distance during this instant, ‘instantaneous velocity’ has no real-world meaning. Velocity implies motion, and motion requires that some distance be traversed in some time interval. And distance (e.g. between two points A and B) requires some measure of space. It is meaningless to speak of something “moving” unless it moves some distance within some amount of time. And what Zeno’s argument shows us is that we cannot measure this distance using the arrow itself. Therefore, in order for the arrow to be in motion, there must be some background against which it moves. When we imagine ourselves as observing the arrow in motion, we ourselves serve as this background for motion. In other words, what is required for motion is that there are two bodies, in which one moves relative to the other. What we must take from this argument is that motion is not absolute; it is relative between bodies.

The fourth argument that we shall consider takes place in a stadium, and involves three bodies, each being composed of four equal discrete parts arranged linearly, and all three bodies being parallel to one another. The first body (denoted as AAAA) is being approached from opposite ends by the other two (denoted as BBBB and CCCC). When BBBB and CCCC meet, the first B will pass *all* of the Cs, and the first C will pass *all* of the Bs, in the same time interval during which the first B will pass only *half* of the As, and the first C will likewise pass only *half* of the As. Thus, if velocity is relative between bodies, then the velocity of BBBB is twice that in

respect to CCCC as it is to AAAA, and the velocity of CCCC is twice that in respect to BBBB as it is to AAAA. Therefore, BBBB is moving at two different velocities at the same time, and so is CCCC. (Baird and Kaufmann, 26-7)

This argument makes nonsense of the notion of absolute velocity in a relative system; it makes no sense to speak of the velocity of an object, except in relation to some other object, and this needn't be the same velocity as in respect to some third object. Thus, even with some background against which to move, we cannot attribute a definite velocity to the object in motion. In other words, velocity is not a *property* of an individual object, but a *relation* between two objects.

Of course, one might suppose that there is some background of space-time independent of the bodies occupying it, whereupon we might fix all of these relations between the bodies, and so attribute velocity to them singly, rather than relatively. This background would be, of course, the aether, an idea that science abandoned some time ago. Alternatively, we could (in principle) determine the relative velocities of all objects in the universe, and mathematically derive some choice point: an object which we could denote as being "at rest", and in relation to which everything else is in motion. But this would be a mere convenience, since there is no background against which we can rightly claim that this object is "at rest", and it could only be so considered in relation to everything else. Instead, we tend to choose other objects that are larger than us, like the earth, or the sun, or some galaxy, by which to fix this relative motion. Here we may be further tempted to suppose that we may thus achieve some notion of absolute motion, since we can fix our movement relative to the earth, whose motion can in turn be fixed relative to the sun, whose motion can be fixed relative to the stars, whose motion can be fixed relative to the other

galaxies... but here we have run out of larger bodies to which we can fix relative motion, since all the galaxies are in motion relative to each other. What then is motion? I know not.

There is a difficulty inherent in the first two arguments that we considered, which we passed over without comment, but are now in a position to address. The first argument had some object A moving toward some object B. We concluded that space and time form one entity: space-time. This allowed us to temporarily bypass the inherent difficulty in dividing up space and time into infinite but discrete parts. However, in the second argument, where A is in motion relative to B, and A and B are *both* in motion relative to C, we arrive at a further difficulty: that there must be a smallest discrete space-time interval. This is not reconcilable with the assumption that space-time is infinitely divisible, which is something that is assumed in the Calculus. Perhaps infinite divisibility is just something that we *assume* in order to arrive at an approximate summation of the smallest discrete units of space-time. This is supported in modern physics by the notions of the Planck length and the Planck time, which are the smallest *meaningful* units of space and time. But this raises yet another difficulty: if space-time comes in discrete units, and it is not meaningful to speak of anything “in between”, then how is motion possible? We cannot rightly say that some object “moves” from one place to another; it is more like teleportation. Of course, this is not how motion looks to us empirically, but looks can be deceiving. What psychologists call “apparent motion”, best exemplified by the Phi-Phenomenon, shows us that such “teleportation” from one place to another, even on large scales, can appear to be continuous motion. Perhaps motion is just all in our heads.

Another approach, to which we are led by the fourth argument, would be to deny any notion of absolute velocity. But where would that leave us? Einstein’s theory of Special Relativity describes such relative motion, and its effects on space-time, but it doesn’t help us

understand what motion *is*. If motion is a relation between bodies, then what is it a relation *of*? Must there not then be some property that we can attribute to bodies individually, on which this relation can supervene? What sort of property would that be? To my knowledge, Einstein never even considered this question, much less proposed an answer to it. Therefore, so far as I can tell, Special Relativity cannot provide us with an answer to the problems posed by Zeno. At this point I am more than willing to concede that there is no such thing as motion. However, I am quite uncertain as to what should replace this forsaken idea.

The fundamental problems raised by Zeno cut to the very core of our notions of space and time. These problems continue to plague us today in the form of the apparent incommensurability between quantum physics and relativity. Neither one provides any solution to our problem, but perhaps the answer would come in the form of yet another theory that could reconcile them. In fact, I dare say, the solution to reconciling quantum physics with relativity is staring us smack in the face, and it is due entirely to Zeno. However, this solution currently evades me, and I can only wish that I had access to Zeno's other arguments, which might shed further light on this problem.

References

Baird, Forrest E. & Walter Kaufmann (2003). *Ancient Philosophy*. New Jersey: Prentice Hall.